

Aliasing effects in digital images of line-pair phantoms

Michael Albert, Daniel J. Beideck, Predrag R. Bakic, and Andrew D. A. Maidment

Citation: Medical Physics **29**, 1716 (2002); doi: 10.1118/1.1493212 View online: http://dx.doi.org/10.1118/1.1493212 View Table of Contents: http://scitation.aip.org/content/aapm/journal/medphys/29/8?ver=pdfcov Published by the American Association of Physicists in Medicine

Articles you may be interested in

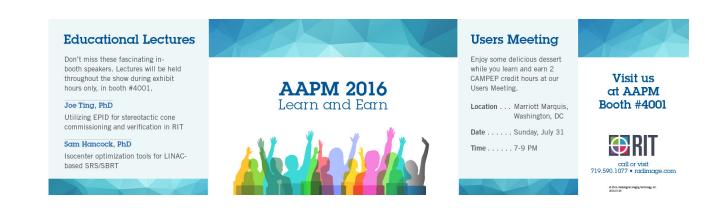
Radiation dose assessment in a 320-detector-row CT scanner used in cardiac imaging Med. Phys. **38**, 1473 (2011); 10.1118/1.3558020

Use of a line-pair resolution phantom for comprehensive quality assurance of electronic portal imaging devices based on fundamental imaging metrics Med. Phys. **36**, 2006 (2009); 10.1118/1.3099559

Tissue-equivalent materials for construction of tomographic dosimetry phantoms in pediatric radiology Med. Phys. **30**, 2072 (2003); 10.1118/1.1592641

Advances in Film Processing Systems Technology and Quality Control in Medical Imaging Med. Phys. **28**, 1813 (2001); 10.1118/1.1388904

Verification of compensator thicknesses using a fluoroscopic electronic portal imaging device Med. Phys. **26**, 1524 (1999); 10.1118/1.598648



Aliasing effects in digital images of line-pair phantoms

Michael Albert, Daniel J. Beideck, Predrag R. Bakic, and Andrew D. A. Maidment^{a)} Department of Radiology, Thomas Jefferson University, Philadelphia, Pennsylvania 19107-5563

(Received 26 December 2001; accepted for publication 16 May 2002; published 18 July 2002)

Line-pair phantoms are commonly used for evaluating screen-film systems. When imaged digitally, aliasing effects give rise to additional periodic patterns. This paper examines one such effect that medical physicists are likely to encounter, and which can be used as an indicator of super-resolution. © 2002 American Association of Physicists in Medicine. [DOI: 10.1118/1.1493212]

Discrete sampling leads to a variety of surprising phenomena such as the stroboscopic effect, Moiré patterns,¹ and the apparent reversal of the motion of wheel-spokes in motion pictures when the angular velocity of the wheel is almost sufficient to return the wheel to its initial position (or an indistinguishable position) in the interval between each frame. In medical physics, the most frequently encountered effect is probably the partial–volume effect² in CT or MR. The increasing use of digital detectors for general radiography is likely to produce more examples of such effects with which a medical physicist should be familiar.³ This paper gives a practical example of an aliasing effect that is easily observed and quantified.

Figure 1 shows a detail of a line-pair phantom (Nuclear Associates CN 4779, Hicksville, NY) imaged in contact with a digital detector^{4,5} (Direct Ray prototype, Hologic, Wilmington, DE) as is conventional for measuring detector limiting resolution. The bar pattern is rotated by a small angle (e.g., 1.93° in Fig. 1) with respect to the detector array. The numerical values shown in the figure represent the spatial frequency in lp/mm according to the manufacturer of the phantom. One of us (D.B.) noticed, superimposed on the expected bar pattern, light and dark bands with a greater spacing and a different orientation than the bar pattern. The superimposed pattern is apparently periodic and the orientation of the bands varies with the fundamental frequency of the bar pattern, passing through the normal to the bar pattern between 3.4 lp/mm and 3.7 lp/mm compared to the detector's cutoff frequency of 3.6 lp/mm.

These observations lead one to interpret the observed effect in terms of an aliasing artifact, as will be explained in detail with reference to Fig. 2. A bar pattern can be represented as a Fourier series of the form

$$\sum_{n=-\infty}^{\infty} a_n e^{2\pi i n(k_x x + k_y y)},\tag{1}$$

where nk_x and nk_y represent the components of the spatial frequency vector of the *n*th harmonic. In Fig. 2, the circles along the diagonal line represent the spatial frequency vectors for the 3.4 lp/mm pattern with $n = \pm 1, \pm 2$. The squares similarly represent the spatial frequencies occurring in the 3.7 lp/mm pattern. The position of the n=0 vector at the origin in frequency space is also shown, and the vertical axis has been exaggerated as otherwise the angle of the bar patterns with respect to the horizontal would not be perceptible. In both cases, the second harmonic (n=2) corresponds to an aliased frequency that is obtained by adding or subtracting 1 lp/pixel=7.2 lp/mm from the horizontal component of the spatial frequency vector, as indicated by the arrows. The aliased frequencies represented by the circles, corresponding to the 3.4 lp/mm pattern, lie in quadrants II/IV, while the aliased frequencies represented by squares, corresponding to the 3.7 lp/mm pattern, have crossed over into quadrants I/III. Bar patterns of somewhat lower frequency, such as 3.1 lp/ mm, would correspond to positions on the diagonal line closer to the origin, and thus the aliased frequencies would be further into quadrants II/IV. Somewhat higher spatial frequencies such as 4.0 lp/mm would correspond to positions further from the origin on the diagonal line and be further into quadrants I/III. In Fig. 3, lines have been added to Fig. 1 to show the spacing between lines of equal phase corresponding to the aliased n=2 components using the measured angle of $\theta = 1.93^{\circ}$ and the nominal spatial frequencies (except for the nominal 3.1 lp/mm pattern, where 3.25 lp/mm shows better agreement with the observed pattern).

While the human eye is generally poor at making quantitative estimates, when looking at a series of bar pattern images it is relatively easy to identify when the spatialfrequency vector of the superimposed pattern passes through

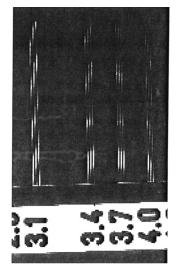


FIG. 1. Detail of a line-pair phantom with window and level adjusted to demonstrate the periodic bands which run on diagonals relative to the primary patterns.

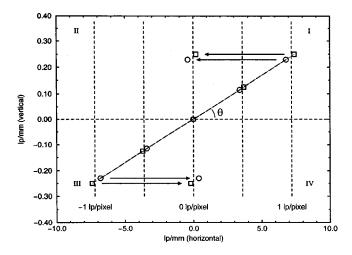


FIG. 2. The line-pair patterns in Fig. 1 are represented in Fourier space by sums of wave functions whose wave vectors (circles 3.4 lp/mm, squares 3.7 lp/mm) lie at regular intervals along a diagonal at an angle θ to the horizontal (in this example $\theta = 1.93^{\circ}$). Arrows show displacement of frequencies by aliasing, vertical lines indicate multiples of the limiting frequency of the detector (i.e., one-half the reciprocal of the sampling pitch).

the vertical. Quantitatively, for a square grid of pixel-pitch a this effect can be observed when the spatial frequency of the line-pair regions vary about a frequency k satisfying

$$nk\cos(\theta) = m/a, \tag{2}$$

where *n* is the harmonic of the line-pair pattern inducing the effect and *m* is a small integer. Observing the effect for a given *m* requires that the detector respond to spatial frequencies of 2m times the nominal limiting frequency of 1/2a, so that the effect is likely noticeable only for m = 1. Similarly it is likely that the effect will only be noticeable for small values of *n*. In Fig. 4 a different region of the line-pair phantom is shown in which the effect is seen for the n=3 harmonic at $\frac{1}{3}$ lp/pixel.

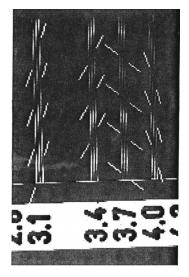


FIG. 3. Superimposed in Fig. 1 are marks whose spacing corresponds to the aliased frequencies as indicated in Fig. 2 for 3.4 lp/mm and 3.7 lp/mm (for the nominally 3.1 lp/mm pattern, 3.25 lp/mm was used).



FIG. 4. Detail of a line-pair phantom with marks superimposed whose spacing corresponds to the aliasing of the third harmonic (for the nominally 2.5 lp/mm pattern, 2.47 lp/mm was used).

The effect discussed here provides a reasonably robust method for identifying which part of the bar-pattern image represents spurious resolution. Referring again to Fig. 1, it is not obvious that the bar patterns at 3.7 lp/mm and above actually exceed the limiting frequency of the detector (and that one is actually seeing an aliased pattern), but the rotation of the superimposed pattern between quadrants is conspicuous. Care must be taken in using this effect, as Fig. 4 shows that the third or higher harmonic can produce similar, although generally weaker, patterns at spatial frequencies below the limiting frequency. Care must also be taken in the angle θ of the bar pattern. The spatial frequency at which the superimposed bands rotate between quadrants is only weakly dependent upon the angle θ for small angles, but the spatial frequency *f* of the superimposed bands themselves will be

$$f = k \sin \theta = \frac{1}{a} \tan \theta \tag{3}$$

at the change in quadrant. For very small angles, the spacing of the superimposed bands becomes larger than the size of the phantom, and thus the superimposed bands may not be visible. The effect of aliasing is still visible, of course, in the change in apparent spatial frequency of the bars themselves. For large angles the spacing of the superimposed bands becomes too narrow to be easily discerned, especially given the fact that these overtone bands have intrinsically less contrast than the primary bars. Typically the bar pattern should be placed at an angle of 1° to 3°.

The effect discussed here requires that the presampled MTF of the detector extend to at least 1/a (twice the nominal limiting spatial frequency). There has been some discussion as to whether detector response to these higher spatial frequencies is useful.^{6,7} Whether or not a system is designed to have an MTF which extends beyond 1/2a, it is important, especially for the medical physicist, to understand how the system is handling those spatial frequencies.

1718 Albert et al.: Aliasing effects in digital images

We would like to thank Dr. Denny L. Y. Lee, Michael Hoffberg, and Cornell Williams for making the Hologic prototype available to us and patiently helping us use it.

- ^{a)}Author to whom correspondence should be addressed. Electronic mail: andrew.maidment@mail.tju.edu
- ¹K. Patorski, *Handbook of the Moiré Fringe Technique* (Elsevier, Amsterdam, 1993).
- ²S. Webb, *The Physics of Medical Imaging* (Hilger, Bristol, Philadelphia, 1988).
- ³M. L. Giger and K. Doi, "Investigation of basic imaging properties in digital radiography. I. Modulation transfer function," Med. Phys. **11**, 287–295 (1984).
- ⁴W. den Boer *et al.*, "Thin film transistor array technology for high performance, direct conversion x-ray sensors," *SPIE Conference on Physics of Medical Imaging*, SPIE **3336** (Bellingham, Washington, 1998), pp. 520–528.
- ⁵D. L. Lee, L. K. Cheung, B. Rodricks, and G. F. Powell, "Improved imaging performance of a 14×17-inch Direct Radiolgraphy(TM) System using Se/TFT detector," *SPIE Conference on Physics of Medical Imaging*, SPIE **3336** (Bellingham, Washington, 1998), pp. 14–23.
- ⁶J.-P. Moy, "Signal-to-noise ratio and spatial resolution in x-ray electronic imagers: Is the MTF a relevant parameter?," Med. Phys. 27, 86–93 (2000).
- ⁷M. Albert and A. D. A. Maidment, "Linear response theory for detectors consisting of discrete arrays," Med. Phys. **27**, 2417–2434 (2000).